Analytical Definition of Aircraft Shape

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Analytical definition of airplane shape in combination with coordinate dimensioning has been shown to be a powerful tool in aircraft production. A very practical method for shape definition has been used at SAAB, Sweden for more than 20 yr. The shape of an aircraft is treated mathematically throughout, and all data from the mathematical definition of shape are delivered in coordinate tables. The outer surface of an aircraft is divided in different segments in a suitable way and the equations of the longitudinal and the sectional boundary curves are chosen to be as simple as possible. In this way anything concerning the aircraft shape which is of importance in production and design can be calculated. The method described below is fully applicable even if a computer is not available.

Introduction

GRAPHICAL methods are commonly used to define the shape of ships and airplanes. These methods require full-scale drawings of the cross sections and longitudinal contour lines of the object on a lofting floor. Production use of the geometrical data obtained in this manner requires a number of retracings, often performed by different persons each time. Thus, tracing errors and errors in measurements can hardly be avoided. The result is poor accuracy, with tolerances no better than 2 mm to be expected in airplane production. At SAAB the graphical method was therefore abandoned at an early stage and replaced by an analytical method whereby the problem can be treated mathematically.

Coordinate Dimensioning

A surface is analytically defined when the equation for the surface, $F(x_n,y_n,z_n) = 0$, has been developed. Here (x_n,y_n,z_n) is a fixed coordinate system in which the coordinate points of the surface are measured.

When developing the shape of an airplane, it is usually practical to define the various parts in separate coordinate systems located in such a manner that they fit the symmetry and orientation of, for example, the wing, the horizontal and vertical tail surfaces, and the fuselage. Since the wing usually is mounted on the fuselage with an angle of incidence, it would be very inconvenient to define it in a coordinate system (x_n,y_n,z_n) that is better adapted to the symmetry of the fuselage.

Instead, a new coordinate system (x_m,y_m,z_m), adjusted to the symmetry of the wing, is introduced, and a much simpler wing equation is obtained. The various coordinate systems must be identified with different subscripts in order to avoid confusion. A point on the wing surface may, if so desired, also be measured in the (x_n,y_n,z_n) system by means of a transformation between the two coordinate systems. The coordinate systems introduced to facilitate the mathematical description of the surface are, of course, equally important as a basis for dimensioning the various design elements of the airplane.[1]

Analytical Shape Definition

Consider, as an example, the problems encountered when attempting to define a part of an airplane fuselage. It is necessary to develop analytical expressions, in principle $F(x,y,z) = 0$, which without serious deviations describe the surface of a preliminary layout, suggested by the project group (see Fig. 1a).

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Each contour line in the body plan may be described by a single function $f(y,z) = 0$ that satisfies a sufficiently large number of conditions to describe the whole line with desired accuracy. However, one soon finds that $f(y,z) = 0$ even for rather simple surfaces becomes complicated and difficult to handle, and often contains unpleasant irregularities such as undesired inflections.

It is therefore practical to divide the loosely sketched surface into several segments that can be defined sufficiently well by means of simple, easy-to-handle functions. This method, which will be described below, has proved to be practical and very efficient. The surface in Fig. 1a has been divided into five segments that are joined at P^{01}, P^{02}, P^{03}, P^{04}, and P^{05}, and each segment may then be defined by a simple second-degree curve, a conic section, whose properties are well-known.

It should be noted here that this type of subdivision is done for mathematical purposes only and has no connection with design considerations such as stringer locations or skin plate sizes. The surface is looked upon as a separate mathematical object and the sole purpose of the subdivision is to obtain the most convenient mathematical expressions. No general rule can be given for this subdivision since it depends too much on the surface shape. In really complicated cases

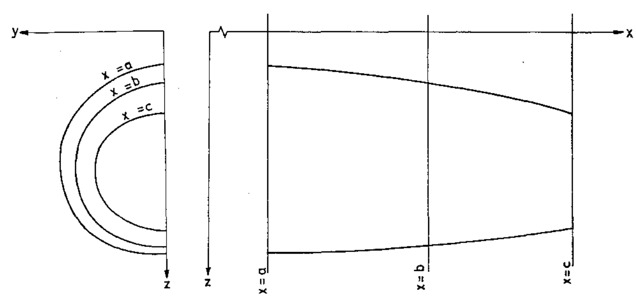

Fig. 1a Preliminary layout of a part of a fuselage.

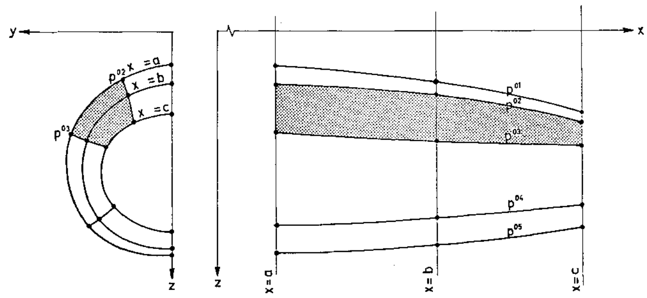

Fig. 1b Segmentation of the layout.

the experience and imagination of the analyst are put to hard tests.

After the division, each segment is treated separately. Consider, for instance, the shaded segment in Fig. 1b. The longitudinal boundary curves are called directrices and are denoted P^{02} and P^{03}. These curves are easiest to define as intersections between two cylinders:

$$P^{02}\begin{cases} y = f_1(x) \\ z = g_1(x) \end{cases} \quad (1)$$

$$P^{03}\begin{cases} y = f_2(x) \\ z = g_2(x) \end{cases} \quad (2)$$

In order to separate the various directrix curves from each other, it is necessary to identify the various running points by

$$\bar{P}^i = (x,s^i,t^i) \quad (3)$$

where i is the identifying index. The following is then obtained:

$$\bar{P}^{02} = [x,s^{02}(x),t^{02}(x)] \quad (4)$$

$$\bar{P}^{03} = [x,s^{03}(x),t^{03}(x)] \quad (5)$$

The tangents of the directrix curves are obtained by differentiating the position vectors with respect to the parameter, in this case x:

$$\frac{d\bar{P}^{02}}{dx} = \left(1, \frac{ds^{02}}{dx}, \frac{dt^{02}}{dx}\right) = (1, \tan\omega_x{}^{02}, \tan\psi_x{}^{02}) \quad (6)$$

$$\frac{d\bar{P}^{03}}{dx} = \left(1, \frac{ds^{03}}{dx}, \frac{dt^{03}}{dx}\right) = (1, \tan\omega_x{}^{03}, \tan\psi_x{}^{03}) \quad (7)$$

As functions $f_1(x)$, $g_1(x)$, $f_2(x)$, and $g_2(x)$ are selected simple algebraic expressions of the second or third degree satisfying the points in the given dimensions on the preliminary design layout.

Next step is to consider the cross-section curve between directrices, section $x = a$, for example. The cross-section curve, from now on referred to as the generator, is replaced by a conic section that passes exactly through the directrix points and, as well as possible, adheres to the original curve shape. As an aid to the determination of the generator a local coordinate system is introduced (Fig. 2).

The following expression is selected for the generator:

$$A(u)^2 + (v)^2 + 2Cu + 2Dv = 0 \quad (8)$$

The constants of this equation must be made to vary with x so that the generator describes the suggested cross-section layouts as x varies from $x = a$ to $x = b$ and further on to $x = c$. Thus, we obtain a surface that is analytically defined, in this case $a \leq x \leq c$ (Fig. 1b).

It is, however, obvious that the aforementioned conic section $A(u)^2 + (v)^2 + 2Cu + 2Dv = 0$ is not completely defined by the two directrix points only. Each directrix must also give the local slopes of the generator segments, called $\alpha_z{}^{02}$ and $\alpha_z{}^{03}$. These slope expressions are called angle directrices and are, of course, also made to vary with x in such a manner that the conic section, the generator, describes the desired surface segment.

We now have three conditions specified along each directrix, and the three constants of the generator may then be determined for any value of x. According to Fig. 2 we have

$$u_1(x) = t^{03}(x) - t^{02}(x) \quad (9)$$

$$v_1(x) = s^{03}(x) - s^{02}(x) \quad (10)$$

$$\tan\alpha_0 = \tan\alpha_z{}^{03}(x) \quad (11)$$

$$\tan\alpha_1 = \tan\alpha_z{}^{02}(x) \quad (12)$$

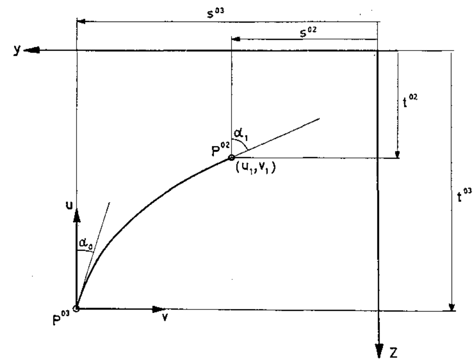

Fig. 2 Local auxiliary coordinate system of a segment.

These conditions satisfy the generator:

$$A(u_1)^2 + 2Cu_1 + 2Dv_1 = -(v_1)^2 \quad (13)$$

$$Au_1 + C + D\tan\alpha_1 = -v_1\tan\alpha_1 \quad (14)$$

$$C + D\tan\alpha_0 = 0 \quad (15)$$

From this system of equations A, C, and D are found:

$$A = -\frac{\left[\tan\alpha_0 - \frac{v_1}{u_1}\right]\tan\alpha_1 + \left[\tan\alpha_1 \frac{v_1}{u_1}\right]\tan\alpha_0}{\left[\tan\alpha^0 - \frac{v_1}{u_1}\right] + \left[\tan\alpha_1 - \frac{v_1}{u_1}\right]}\cdot\frac{v_1}{u_1} \quad (16)$$

$$C = -D\tan\alpha_0 \quad (17)$$

$$D = -\frac{\tan\alpha_1 - (v_1/u_1)}{[\tan\alpha_0 - (v_1/u_1)] + [\tan\alpha_1 - (v_1/u_1)]}\cdot v_1 \quad (18)$$

According to Fig. 2, the transformation that relates the local $(u - v)$ system to the main system is given by

$$y = s^{03} - v \quad (19)$$

$$z = t^{03} - u \quad (20)$$

The equation for the surface segment is now obtained by introducing (19) and (20) in (8):

$$A(x)\cdot[t^{03}(x) - z]^2 + [s^{03}(x) - y]^2 + 2C(x)\cdot[t^{03}(x) - z] + 2D(x)\cdot[s^{03}(x) - y] = 0 \quad (21)$$

or written in a simple way, $F(x,y,z) = 0$. It is now possible to calculate an arbitrary section in the defined surface, $x = p$, by first finding the conditions for the generator at $x = p$ from the directrix equations and then introducing the found values in the equations for the generator constants.

The remaining surface segments $P^{01} \rightarrow P^{02}$, $P^{03} \rightarrow P^{04}$, and $P^{04} \rightarrow P^{05}$ are treated in the same way as $P^{02} \rightarrow P^{03}$ and when all segment equations have been found, the whole surface has been defined. Hence, the problem of defining a surface has been largely reduced to a problem of determining the "directrix skeleton" for the moving generators.

Conic-section generators are used in order to avoid undesired inflections in the surface. If, however, an inflection is wanted, it is convenient to lay the directrix curve through the inflection points so that the desired inflection, and only this, is obtained.

The problem of developing the expressions for the directrices is solved the same way as for the generators. Instead of using difficult polynoms of higher degrees with many terms, it is even here an advantage to subdivide and use a number of simple expressions, conic sections for instance, which are successively changed in order to describe the desired line.

The cross-section segmentation and the simple generator function together with the selection of suitable directrix functions for the segment ends make it possible to define even complicated surfaces in a simple way, since the directrix skeleton defines geometrical characteristics in the surface itself.

It is not necessary to compare the resulting surface with a previously drawn body plan, since the analytical definition is made directly as specified by the aerodynamicist and the designer. Neither is it necessary to make graphical layouts of the cross sections, especially not before the analytical definition of the shape.

A surface defined in this manner may be differentiated in each point; i.e., the surface has neither corners nor edges. It is, however, possible to obtain both corners and edges by suitable selection of directrices.

Applications of Analytical Shape Definition

The analytical method makes it possible to calculate an unlimited number of cuts through a surface. It also eliminates all lofting errors, and the exactness can be made good enough to suit all practical purposes. Furthermore, through the use of coordinate tables for data distribution it is possible to avoid errors due to faulty tracing and incorrect measurements, since all people concerned easily may be supplied with their own set of tables.

When a surface $F(x,y,z) = 0$ has been defined, it is possible to obtain the surface normal $\bar{N}$ in an arbitrary point $\bar{N} = \text{grad } F$ and thereafter any parallel surface may easily be calculated. A parallel surface at the distance t from $F = 0$ is obtained by measuring the distance t along the normal $\bar{N}$ from each point at $F = 0$. In airplane production it is very important to be able to calculate these parallel surfaces, since the skin thickness may vary in different parts of an airplane component. Furthermore, by means of the normal, it is possible to determine the angles between rib anges and surfaces etc.

Analytical shape definition is well-suited for aerodynamic calculations. Area distribution can easily be calculated both in the transonic and the supersonic case. The wetted area as well as the pressure distribution can also be calculated.

Final Remarks

The analytical definition of surfaces described in this article was originally developed by Nils Lidbro. The method has been used at SAAB with complete success for more than twenty years. Analytical definition of shape may of course also be used for objects other than airplanes, as, for instance, for ships.[2] The hull of a ship is usually much simpler to define than the surface of an airplane, and the work involved in analytical definition of a ship hull is therefore much less than that for an airplane.

In connection with this it may be worth mentioning that the so-called "three secrets of the shipwrights" used for determination of the shape of ship hulls in the sixteenth and seventeenth centuries are largely the same as the analytical definition of shape used at SAAB. However, these shipbuilders did not use generators more advanced than straight lines and circle segments. The directrix skeleton was determined by the specifications for the radius variation and the location of the centers of the generator circles.

References

[1] Lidbro, N., "Modern aircraft geometry," Aircraft Eng. **28,** 388–394 (November 1956).

[2] Lidbro, N., "Analytische Formbestimmung von Schiffen," Shiffstechnik **8,** 91–96 (July 1961).